# Spatial forecast verification with wavelets

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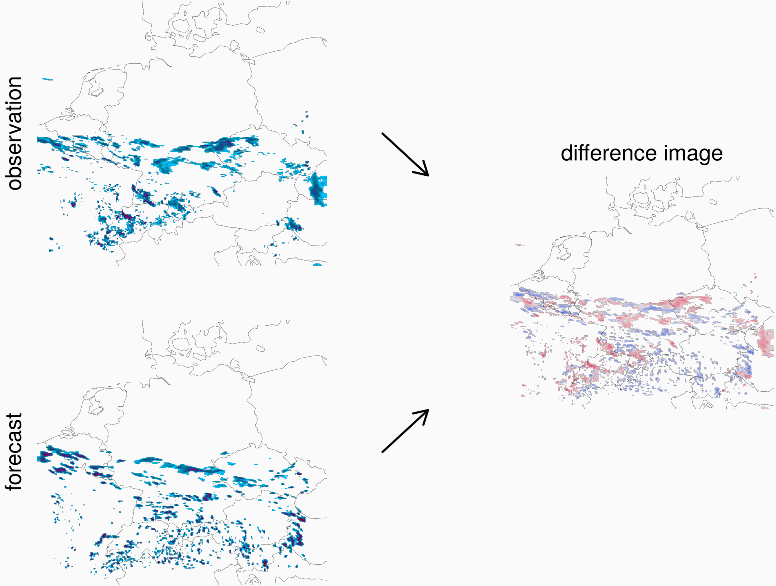


*Double penalty:* Displacement punished twice

- Use wavelets to extract a field’s structure
- Verify structure, disregard location
- Test the procedure using synthetic rain fields

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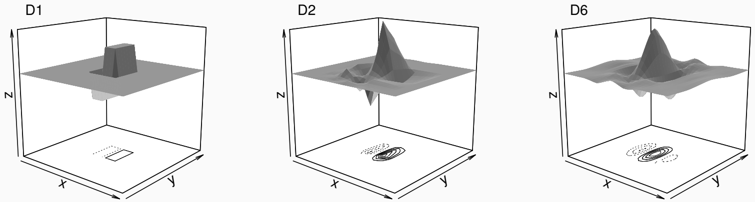
# Method

# The 2D redundant discrete wavelet transform




Transform field to a new basis, generated by a mother wavelet $\psi(\mathbf{r})$

Rotation: $\psi_{\text{vert}}(\mathbf{r}), \psi_{\text{horiz}}(\mathbf{r}), \psi_{\text{diag}}(\mathbf{r})$

Translation: $\psi_{\mathbf{u}}(\mathbf{r}) = \psi(\mathbf{r} - \mathbf{u})$

Scaling: $\psi_j(\mathbf{r}) = \psi(\mathbf{r}/2^j)$

$\rightarrow$ daughter wavelets $\psi_{dir,\mathbf{u},j}(\mathbf{r})$

# Procedure

1. $2^J \times 2^J$ intensities $\xrightarrow[\text{transform}]{\text{redundant}}$ $2^J \times 2^J$ local wavelet-spectra

`www.youtube.com/embed/4fEkw76GXLk?autoplay=1`

# Procedure

1. $2^J \times 2^J$ intensities $\xrightarrow[\text{transform}]{\text{redundant}}$ $2^J \times 2^J$ local wavelet-spectra

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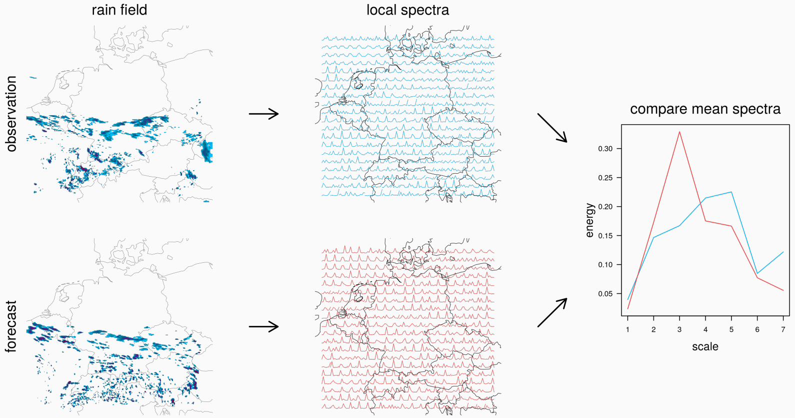
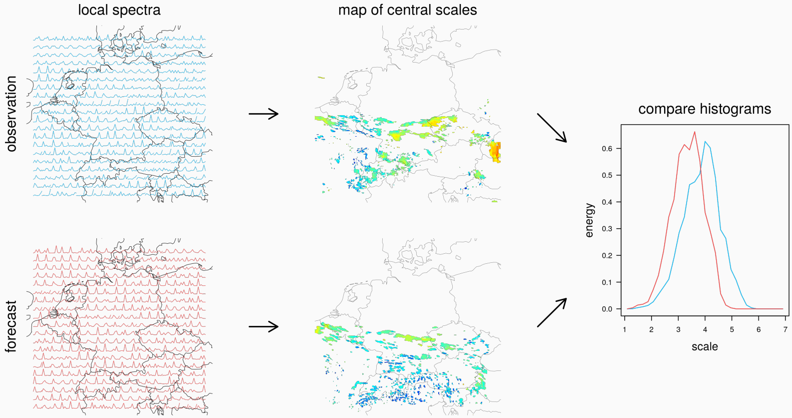
2. Average over the three directions (*for now …*)
3. remove bias towards large scales (Eckley et al. 2010), normalize
4. aggregate in space … *but how?*
5. verify … *but how?*

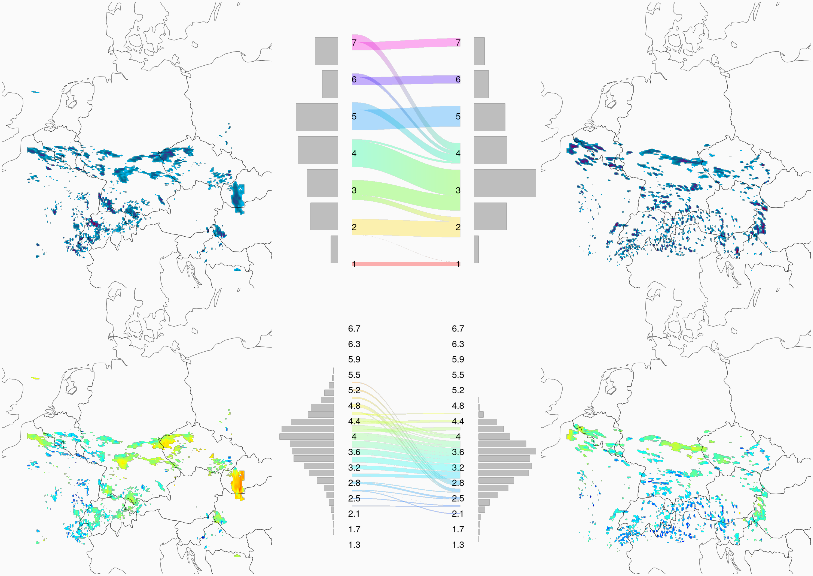
# Aggregation #1: Spatial mean spectra



# Aggregation #2: Map of central scales



# Verification: Earth Mover's distance




EMD bounded from below by the difference in centre → obtain sign

# Random rain fields

# A stochastic model for precipitation

Hewer (2018): "Stochastisch-physikalische Modelle für Windfelder und Niederschlagsextreme"

$$\text{Precip} = \max\left( \underbrace{E}_{\textit{evap.}} - \underbrace{T}_{\textit{thresh.}} - \underbrace{\mathbf{v}\cdot\nabla q}_{\textit{advection}} - \underbrace{q\nabla\cdot\mathbf{v}}_{\textit{convergence}},\ 0 \right),$$

$$\mathbf{v} = \nabla \times \Psi + \nabla\chi$$

$$\Psi, \chi, q \sim \mathcal{N}\left(\mathbf{0},\ \text{Matérn}(\ ||\underbrace{b}_{\text{scale}}(\mathbf{t}-\mathbf{s})||,\ \underbrace{\nu}_{\text{smoothness}}\ )\right)$$

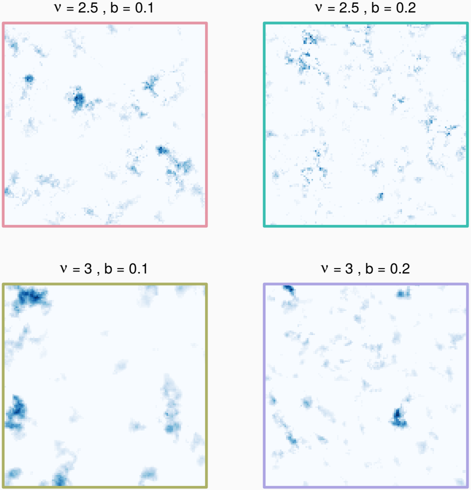
large $\nu \rightarrow$ smooth fields

large $b \rightarrow$ small features

# Simulated rain fields

rough &
large scale

rough &
small scale

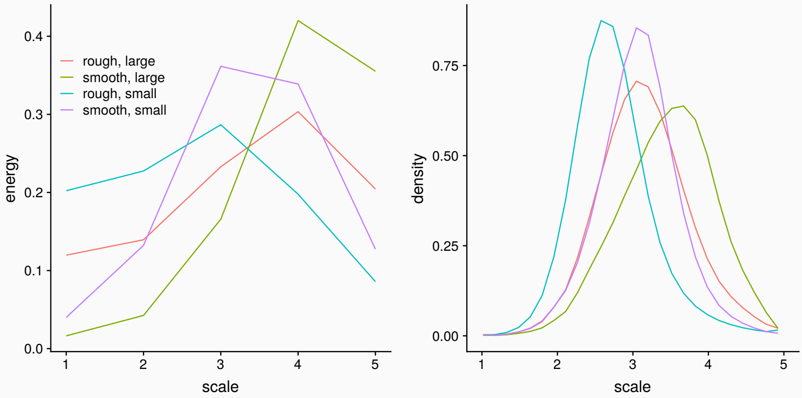



smooth &
large scale

smooth &
small scale

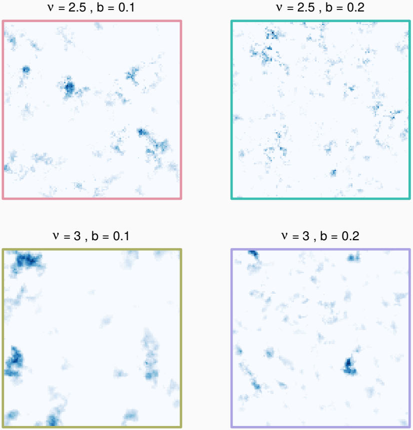
# Simulated rain fields

Average spectra and histograms over many realizations:



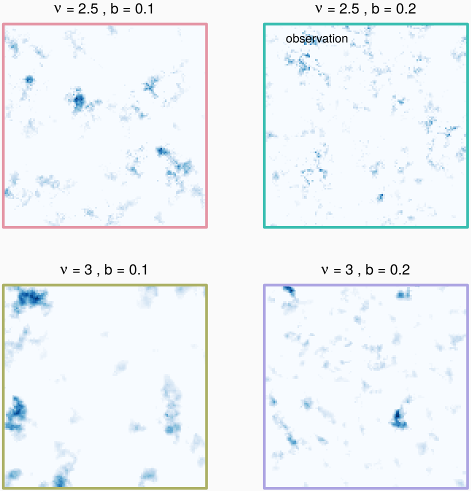
# Idealized experiments

# Experiment set-up




1. Draw an **“observation”** from one of the models
2. Draw one **“forecast”** from each model
3. Calculate **local wavelet spectra**
4. Verification using Earth mover’s distance and distance in centre
5. Repeat **1000 times**

# Observations are rough and small-scaled



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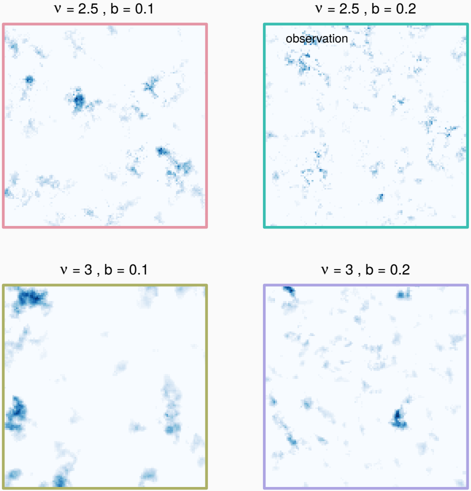


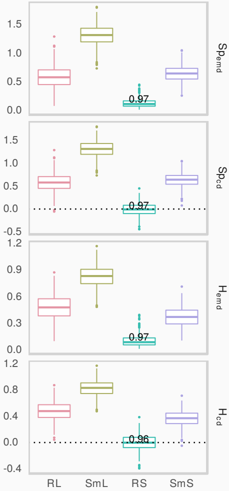

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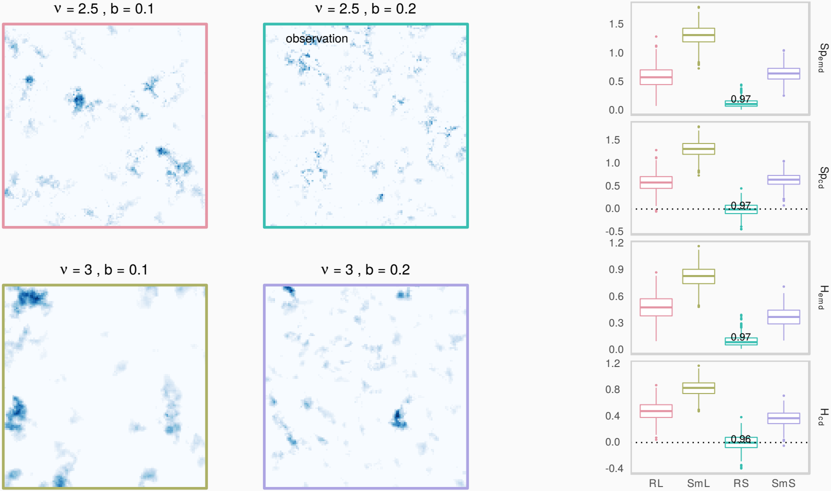
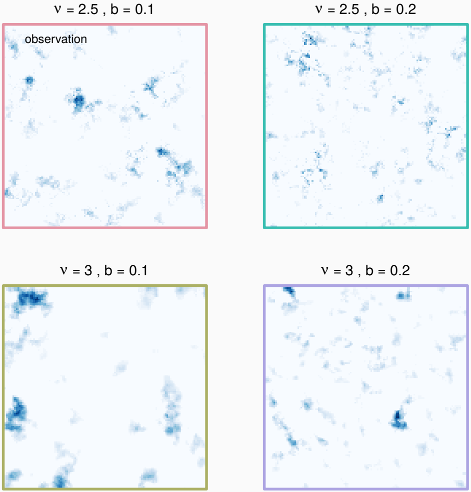



→ correct forecast gets best scores, all others are too large-scaled

# Observations are rough and large-scaled



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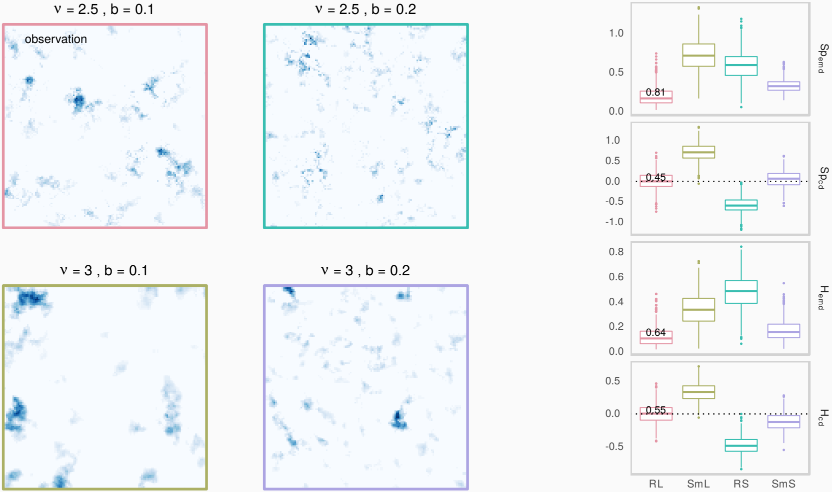

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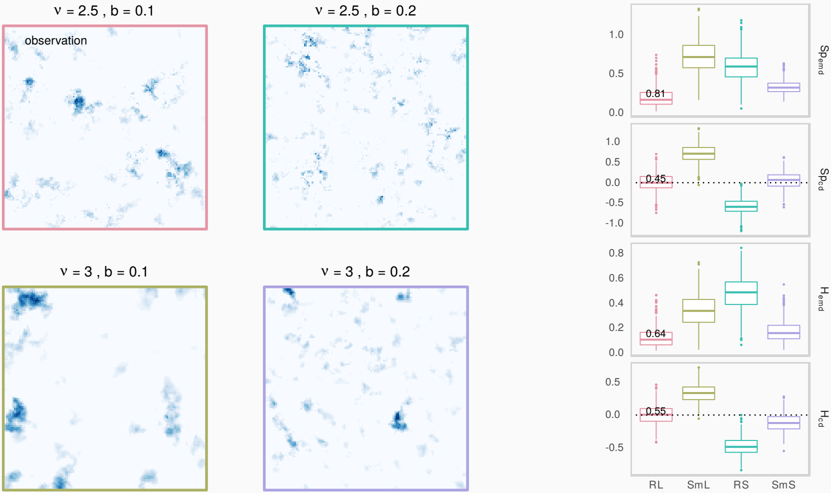
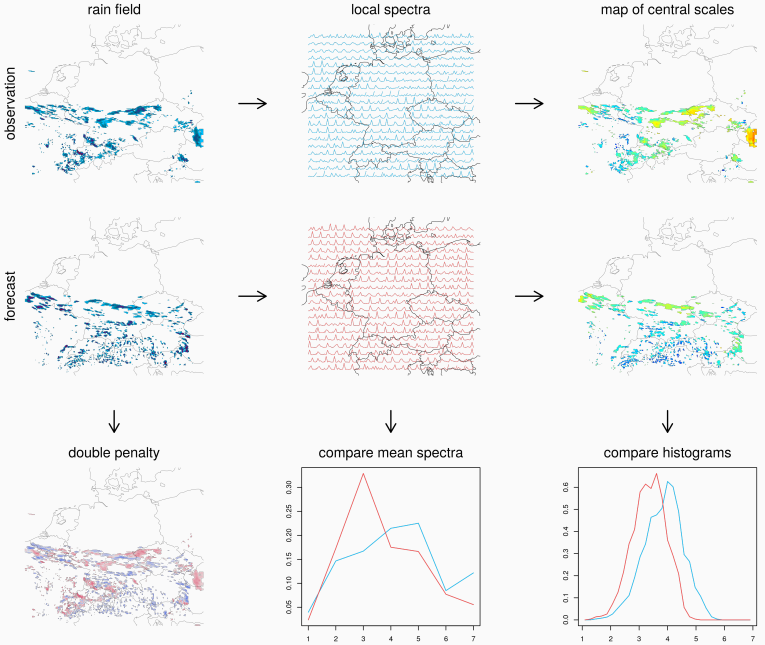



→ higher failure-rates, $(\nu = 2.5, b = 0.1)$ similar to $(\nu = 3, b = 0.2)$

# Summary and outlook


rain field
local spectra
map of central scales
observation
forecast
double penalty
compare mean spectra
compare histograms

# Summary

- Wavelets extract the structure of rain fields
- Two ways of aggregating the information:
  Spatial mean and central scales
- Physically consistent random test cases:

Code for the random rain fields and wavelet-based verification:
`https://github.com/s6sebusc/wv_verif`

Buschow, S., Pidstrigach, J., and Friederichs, P.: *Assessment of wavelet-based spatial verification by means of a stochastic precipitation model (wv_verif v0.1.0)*, Geosci. Model Dev. Discuss., `https://doi.org/10.5194/gmd-2019-90`, in review, 2019.

# Outlook

- Systematic study of real cases
- Include direction information
- Other variables are also possible

# References


- Idris A Eckley, Guy P Nason, and Robert L Treloar. "Locally stationary wavelet fields with application to the modelling and analysis of image texture". In: *Journal of the Royal Statistical Society: Series C (Applied Statistics)* 59.4 (2010), pp. 595–616.
- Rüdiger Hewer. "Stochastisch-physikalische Modelle für Windfelder und Niederschlagsextreme". PhD thesis. University of Bonn, 2018.